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Stramigioli, S.; Secchi, C.; Schaft, A.J. van der; Fantuzzi, C.

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A Novel Theory for Sample Data System Passivity

S.Stramigioli¹, Member IEEE, C.Secchi², Student Member IEEE,
 A.J. van der Schaft¹ Fellow IEEE, C. Fantuzzi², Member IEEE

¹Drebbel Institute, Univ. of Twente
 P.O.Box 217, 7500 AE Enschede, NL
 S.Stramigioli@ieee.org,
 A.J.Vanderschaft@math.utwente.nl

²DISMI, Univ. of Modena and Reggio Emilia
 Viale Allegri 13, 42100 Reggio Emilia, Italy
 secchi.cristian@unimo.it
 fantuzzi.cesare@unimo.it

Abstract

This paper presents a novel way to approach the interconnection of a continuous time and a discrete time physical system. This is done in a way which preserve passivity of the coupled system independently of the sampling time T . A direct application is in the field of haptic displays where virtual environment should feel like equivalent physical systems.

1 Introduction

Several researches and case studies have shown that stability is a key issue in the implementation of an haptic display. Oscillations or unstable behaviors in the haptic device lead to unnatural feedback from the virtual environment or could even cause hazardous situations for the human operator.

The stability analysis of the haptic device is not a trivial task and is hardly solvable with parameter based tools of non-linear control. As a matter of fact, interesting virtual environments are very often non-linear and the dynamic of the human operator, which has a non-negligible role in the haptic chain, is difficult to model.

A very suitable tool to ensure the stability of the haptic display is passivity theory. It has been shown that it is sufficient to grant the passivity of a system in order to have a stable behavior (van der Schaft 2000). Moreover we can apply these concepts both to linear and nonlinear systems. On the other side, Hogan (Hogan 1989) has shown that the behavior of the human operator is passive in the range of frequencies of interest in haptics.

A rigorous examination of the stability of an haptic display has been made by Colgate (Colgate *et al.* 1993) who showed which is the necessary amount of damping to ensure the passivity of the system, once a model of the virtual environment and the sampling

time are given. More recently, he also introduced the idea of *virtual coupling* (i.e. a way of decoupling the haptic device control problem from the virtual environment) that makes possible to guarantee passivity for arbitrary passive operators and environments (Colgate and Schenkel 1994) and even for a class of non passive environments (Miller *et al.* 2000).

Adams (Adams and Hannaford 1999) developed a method based on two-port theory and Llewellyn's criterion to design the virtual coupling for both impedance and admittance causalities of the haptic display.

A possible drawback of the fixed parameters virtual coupling lies in performance decreasing, which is related to an excessive energy dissipation in some working condition. In (Hannaford 2000) Hannaford introduced a virtual coupling strategy with variable parameters, called PO/PC. This strategy uses, loosely speaking, a variable damper that is activated only when an energy increment is detected.

Through this paper, we model the haptic display using energetic interconnection of systems as shown in Fig.1. This scheme shows the energetic exchange among the system components by means of the bond-graph formalism (Stramigioli 2001) with the double vertical bar indicates an exchange of energy which occurs in discrete-time. The element denoted with *SH* represents the sample & hold component, which implements the gate between the continuous and discrete domains.

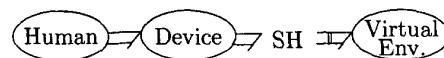


Figure 1: Energetic representation of a haptic display

In the scheme there are two points in which we can have energy generation instead of simple energy ex-

change. The production of extra energy can lead to a non passive behavior and therefore to haptic device instability. The factors which contributes to the production of extra energy have been called “energy leaks” by Gillespie and Cutkosky (Gillespie and Cutkosky 1996). We can mainly distinguish two energy leaks:

- Zero Order Hold
- Discrete Virtual Environment

The zero order hold can represent an energy leak because it keeps a power variable to a constant value during the sample period, regardless of the actual value of its conjugate variable whose behavior could be such to introduce extra energy into the system. It can also happen that a virtual environment, obtained by discretising its continuous passive counterpart, does not preserve passivity. Thus, the discrete virtual environment must be designed carefully in order to avoid a non passive behavior.

The aim of our work is to use the port-Hamiltonian formalism (Courant 1990, Stramigioli 2001) in order to describe the various parts of an haptic interface and their energetic interconnection.

Since each port-Hamiltonian system represents a passive system if the Hamiltonian is bounded from below, we seek for a discretisation method for these systems that preserves their passivity property. On the other side, as every lumped physical system can be represented by a port-Hamiltonian, we can treat a very large number of possible virtual environments with our methodology.

Furthermore, we will try to give an energetic interpretation of the Sample & Hold device in order to design an energetically consistent (and therefore without any extra energy production) interconnection between continuous and discrete port-Hamiltonians.

The paper is organized as follows: in Sec.2 we give some background on the port-Hamiltonian formalism, then in Sec.3 we illustrate a discretisation technique which preserves the passivity of port-Hamiltonian systems. In Sec.4 we describe the concept of sampled data passivity in an energy consistent framework and in Sec.5 we show how it is possible to interconnect discrete and continuous port-Hamiltonian in a passive way. Finally in Sec.6 some simulations validating the theoretical results are presented.

2 Background

In this section we try to give an intuitive description of port-Hamiltonian systems using coordinates in order to concentrate on the prime contribution of the paper. More formal descriptions can be found

in (van der Schaft 2000). We can consider a port-Hamiltonian system as composed of a state manifold \mathcal{X} , an Energy function $H : \mathcal{X} \rightarrow \mathbb{R}$ corresponding to the internal energy, a network structure $D(x) = -D(x)^T$ whose graph has the mathematical structure of a Dirac structure, which is in general a state dependent power continuous interconnection structure, and an interconnection port represented by an effort-flow pair $(e, f) \in V^* \times V$ which is geometrically characterized by dual vector elements. This port is used to interact energetically with the system. The power supplied through a port is equal to $e(f)$ or using coordinates to $e^T f$. We can furthermore split the interaction port in more sub-ports, each of which can be used to model different power flows. We will indicate with the subscript I the power ports by means of which the system interacts with the rest of the world, with the subscript C the power ports associated with the storage of energy and with the subscript R the power ports relative to the dissipative part. Summarizing, we have:

$$\begin{pmatrix} e_I \\ f_C \\ e_R \end{pmatrix} = D(x) \begin{pmatrix} f_I \\ e_C \\ f_R \end{pmatrix}$$

where

$$D(x) := \begin{pmatrix} D_I & G_1 & G_2 \\ -G_1^T & D_C & G_3 \\ -G_2^T & -G_3^T & D_R \end{pmatrix}$$

and D_I, D_C, D_R are skew-symmetric. Due to the skew-symmetry of $D(x)$, we clearly have, using coordinates:

$$P_I + P_C + P_R := e_I^T f_I + e_C^T f_C + e_R^T f_R = 0 \quad (1)$$

which is a power balance meaning that the total power coming out of the network structure should be always equal to zero.

A dissipating element of the system can be modeled using as characteristic equations $e_R = R(x)f_R$ with $R(x)$ a symmetric and positive semi-definite tensor. This implies that

$$f_R = (D_R - R)^{-1} G_2^T f_I + (D_R - R)^{-1} G_3^T e_C$$

and therefore

$$\begin{pmatrix} e_I \\ f_C \end{pmatrix} = \begin{pmatrix} B & A \\ C & D \end{pmatrix} \begin{pmatrix} f_I \\ e_C \end{pmatrix} \quad (2)$$

where:

$$B := D_I + G_2(D_R - R)^{-1} G_2^T \quad (3)$$

$$A := G_1 + G_2(D_R - R)^{-1} G_3^T \quad (4)$$

$$C := -G_1^T + G_3(D_R - R)^{-1} G_2^T \quad (5)$$

$$D := D_C + G_3(D_R - R)^{-1} G_3^T \quad (6)$$

If we furthermore set $\dot{x} = f_C$ and $e_C = \frac{\partial H}{\partial x}$, due to the previous power balance we obtain:

$$\dot{H} + f_R^T R(x) f_R = -e_I^T f_I$$

which clearly says that the supplied power $-e_I^T f_I$ equals the increase of internal energy plus the dissipated one.

3 Discrete Port Controlled Hamiltonian Systems

For a lot of useful applications like haptics, it is meaningful to find a discrete time representation of a physical system which is used as a virtual environment. In this section we will show how to discretise a port-Hamiltonian system preserving its passivity.

We can describe a discrete time port-Hamiltonian system as a continuous time port-Hamiltonian system in which the port variables are frozen for a sample interval T . In what follows we indicate with $v(k)$ the value of the discrete variable $v(t)$ corresponding to the interval $t \in [kT, (k+1)T]$.

If we rewrite Eq.(1) for the discrete case, we have:

$$e_I^T(k) f_I(k) + e_C^T(k) f_C(k) + e_R^T(k) f_R(k) = 0 \quad (7)$$

Furthermore, during the interval k , we have to consider a constant state $x(k)$ corresponding to the continuous time state $x(t)$. This implies that during the interval k , the dissipated energy will be equal to $T f_R^T(k) R(x(k)) f_R(k)$ and the supplied energy will be equal to $-T e_I^T(k) f_I(k)$. In order to be consistent with the energy flows, and as a consequence conserve passivity, we need therefore a jump in internal energy $\Delta H(k)$ from instant kT to instant $(k+1)T$ such that:

$$\Delta H(k) = -T f_R^T(k) R(x(k)) f_R(k) - T e_I^T(k) f_I(k)$$

This implies that the new discrete state $x(k+1)$ should belong to an energetical level such that:

$$H(x(k+1)) = H(x(k)) + \Delta H(k)$$

We can indicate the set of possible energetically consistent states, which can be found solving the previous equation in $x(k+1)$, as

$$I_{k+1} := \{x \in \mathcal{X} \text{ s.t. } H(x) = H(x(k)) + \Delta H(k)\}.$$

Furthermore, from the discrete equivalent of Eq.(2), we have that:

$$f_C(k) = C f_I(k) + D e_C(k) \quad (8)$$

and therefore, for consistency with the continuous dynamics in which $f_C(t) = \dot{x}(t)$, the next state $x(k+1)$ should be such that:

$$f_C(k) = \lim_{T \rightarrow 0} \frac{x(k+1) - x(k)}{T} \quad (9)$$

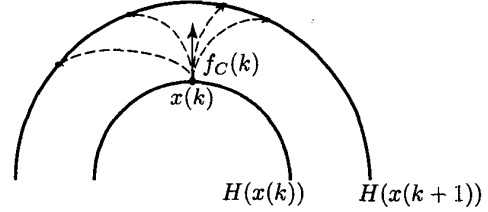


Figure 2: The various possible connections for the state jump

where we considered the definition of the right derivative. The set I_{k+1} can be either empty or have more solutions.

3.1 Case $I_{k+1} \neq \emptyset$

This situation is the most common and corresponds to the normal one. In this case a choice should be made among the possible states of I_{k+1} . Clearly, the state should be in some sense 'close' to the current state $x(k)$ and such that the condition of Eq.(9) is satisfied. A picture which shows graphically the basic idea is reported in Fig.2. The possible curves going through $x(k)$ and having as a tangent $f_C(k) \in T_{x(k)}\mathcal{X}$, could be characterized as geodesics once an affine connection would be defined on \mathcal{X} which are indicated as dotted lines in the figure. In this paper we have considered Euclidean coordinates and an Euclidean connection. In this case, the next state $x(k+1)$ is chosen as the intersection of I_{k+1} with the straight line passing from $x(k)$ and directed along $f_C(k)$. Future research will investigate how to define a connection in such a way to have discrete dynamics that best approximate the continuous ones, as partially explained in (Gonzalez 1996).

3.2 Case in which $I_{k+1} = \emptyset$ and energy leap

This can happen in two situations a) required decrease of energy close to a local minimum or b) required increase of energy close to a local maximum. The case b) does not preclude passivity and therefore will not be analyzed in this paper.

In the situation a) let us indicate with x_{min} one of the states for which the energy has locally a minimum close to $x(k)$ and equal to $H(x_{min})$. This situation is therefore obtained if:

$$\Delta H(k) < H(x_{min}) - H(x(k)).$$

In this situation, it is clearly not possible to find a state $x(k+1)$ compatible with the energy change $\Delta H(k)$ since $I_{k+1} = \emptyset$. If we chose $x(k+1) = x_{min}$ we would implement the smallest error in the energy change, but this would not be a good choice for two main reasons: first, this could create a 'dynamic deadlock' since, in this situation the effort generated by the

energy function and equal to $e_C = \frac{\partial H}{\partial x}$ would be equal to zero and in case no damping would be present, it would be possible to see that this would prevent any further supply of energy from the interconnection port (e_I, f_I) since e_I could be equal to zero and therefore any further change of the state would be impossible. Second it would not help the system to behave in such a way that its dynamic would make possible to correct the energetic discrepancy created since the required $\Delta H(k)$ cannot be performed. A solution to these two problems can be found in what we call *energy leap*. Instead of choosing as a new state x_{min} , we choose as a state $x(k+1)$, a state with the same energy level, but 'symmetrically positioned' with respect to the point of minimum energy x_{min} . This rather fuzzy statement could be made precise once an affine connection would be defined on the state manifold. As already said, considering Euclidean coordinates, it is possible to define the next state as the state having the same energy and lying on a straight line passing through x_{min} and $x(k)$.

Clearly, by construction, we obtain an error in the energy change equal to $\Delta H(k)$ which corresponds to the amount of energy which we supplied to the 'rest of the world' through the power port (e_I, f_I) . On the other hand, by the change of sign in the gradient of the energy function, we practically passed through the minimum in one go and the system will therefore now try to absorb energy from the port (e_I, f_I) . This will then yield an increase of the internal energy. It would therefore then be possible to increase the internal energy not exactly with the required amount but with inferior values. In such a way, it would be possible using energy error book-keeping to maintain passivity of the discrete system. The major point is that it is possible to keep track exactly of the error in the energy values in such a way that they can be then compensated.

As a summary of the procedure just outlined, we hereafter algorithmically explain the way the discrete system can be integrated

1. Given an initial state $x(k)$, we set $e_C(k) = \frac{\partial H}{\partial x}(x_k)$.
2. Using the value of the system input $f_I(k)$ and the previously calculated $e_C(k)$, we can calculate $e_I(k)$, the output of the interaction port, and $f_C(k)$ using the discrete representation of Eq.(2)
3. $f_C(k)$ is then used to calculate the next state $x(k+1)$ using the procedure explained at the beginning of this section.

4 Energy Consistent Sampled Passivity

Consider the port interconnection of a continuous time Hamiltonian system H_C and a discrete Hamil-

tonian system H_D (but the result of this section is independent of the nature of the energetically interconnected systems) through a sampler and zero-order hold as shown in Fig.1. Suppose that H_C has an admittance causality (effort in/flow out) and therefore H_D has an impedance causality (flow in/effort out).

During the dynamic evolution of the two systems between time kT and $(k+1)T$, where T is the sampling time and k is a positive integer, the effort supplied to H_C by H_D will be constant due to the zero-order hold assumption. We will indicate this value as $e_d(k)$. If we indicate the power port at the continuous side with $(e(t), f(t))$, we clearly have:

$$e(t) = e_d(k) \quad t \in [kT, (k+1)T]$$

By looking at the energy flow toward the continuous system, we can see that, if we indicate with $\Delta H_C^{\text{in}}(k)$ the energy which flows through the input power port from time kT up to time $(k+1)T$, we obtain:

$$\begin{aligned} \Delta H_C^{\text{in}}(k) &= \int_{kT}^{(k+1)T} e_d^T(k) f(s) ds = \\ &= e_d^T(k) \int_{kT}^{(k+1)T} f(s) ds = \\ &= e_d^T(k) (x((k+1)T) - x(kT)) \end{aligned} \quad (10)$$

where we indicated with $x(\cdot)$ the integral of the continuous time flow $f(t)$.

Remark 1 *It is important to realize that, in most of useful mechanical applications like haptics, $e_d(k)$ will correspond to forces/moments that a controller would apply to an inertial element. In this case, $x(\cdot)$ would be nothing else than a position measurement of the masses the controller pushes on.*

It is now straight forward to state the following theorem

Th. 1 (Sample Data passivity) *If in the situation sketched before, we define for the interconnection port of H_D*

$$f_d(k) := \frac{x(kT) - x((k+1)T)}{T}, \quad (11)$$

we obtain an equivalence between the continuous time and discrete time energy flow in the sense that for each n :

$$\sum_{i=1}^n e_d^T(i) f_d(i) = - \int_0^{nT} e^T(s) f(s) ds \quad (12)$$

Remark 2 *It is important to notice that the exact equivalence is achieved only by the definition of*

Eq.(11) in which $x(\cdot)$ is usually the easiest variable to be measured in real applications. The negative sign appearing in Eq.(12) is consistent with the fact that the power flowing into the continuous system is minus the power flowing into the discrete side.

5 Passive Coupled Behavior

From the previous considerations, it is possible to understand that at each sampling time, we have an EXACT matching between the physical energy going to the continuous time system and the virtual energy coming from the discrete time port independently of the sample time T and its intersampling behavior. It is remarkable that the choice reported in Eq.(11) which is very simple, normally used and at the same time attractive due to the fact that it corresponds to position measurements, in practice gives such a powerful and at the same trivial result.

This means that we can passively interconnect the two systems in such a way that *independently* of the sampling time and its relation with the characteristics of the interconnected systems, the two systems would be energetically consistent at each sampling time and no energy would be created by the sampling and hold procedure.

The only energy leakage is due to the fact that the discrete time system has no way what so ever to predict the value of the continuous time system at the interconnection port and this implies that only at the end of the sample period will have an exact measure of the energy it supplied to the continuous time system. This gives rise to the problems reported in Sec.3.2. These are structural problems, but they can be either compensated by a clever book-keeping of the energy in excess supplied to the continuous time system, acting directly on the updating law of the state of the discrete system, or by a continuous time damping circuit.

6 Simulations

In this section we will provide two kind of simulations in order to validate our results. In all simulations we deal with the energy leak illustrated in Sec.5 dissipating the extra energy produced. Simulations are necessarily discrete, but we can simulate a continuous system using a sample time much smaller than the one we are using to simulate a discrete system. In such a way we can simulate the interaction between continuous and discrete systems.

In the first kind of simulations we simulate a mass-spring system where the mass is a continuous system and the spring is implemented as a discrete port-Hamiltonian obtained as described in Sec.3. The mass and the spring are connected as illustrated in Sec.5.

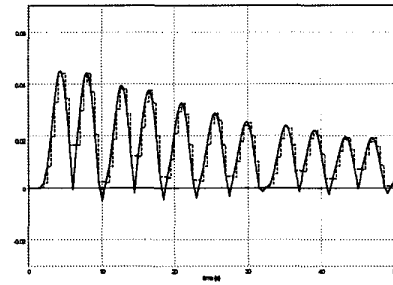


Figure 3: Energy of the continuous power port (continuous line) and of the discrete power port (dashed)

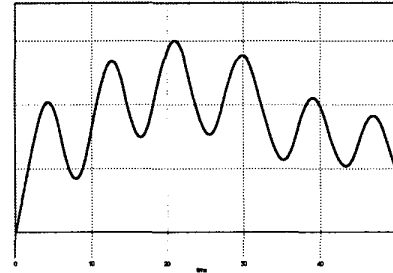


Figure 4: Position of the mass

The mass has an initial state and, therefore, it oscillates around the equilibrium point of the discrete spring.

In the first simulation the sample time is set to $T = 0.5 \text{ sec}$ and we can see in Fig.3 the energetic behavior of the power ports and in Fig.4 the position of the mass. As described in Sec.4, the energies of the continuous and discrete power port, connected by means of the Sample & Hold, match in the sample times. We can notice that the behavior of the system is quite different from the one we would have if we used a continuous spring. In the next simulation we set the sample time at $T = 0.1 \text{ sec}$ and we can see from Fig.5 that in this case the behavior of the system is much more similar to the one of its continuous counterpart. Decreasing the sample time, the behavior of the system gets closer and closer to the one of its continuous counterpart but we can *always* grant the stability of the overall system disregarding the sample time.

The next kind of simulation is an haptic application of our scheme. We have a mass, a continuous haptic device, and a discrete virtual environment, a virtual wall. We implemented the virtual wall as a discrete port-Hamiltonian made up of a parallel of a very stiff spring ($k = 1000 \text{ N/m}$) and of a damper ($b = 30 \text{ Nsec/m}$). The simulation is one-dimensional:

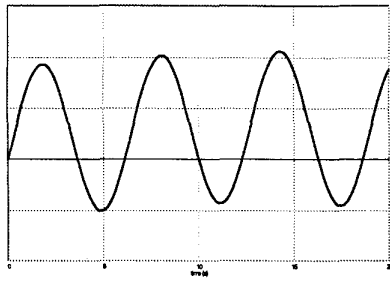


Figure 5: Position of the mass

the wall is at the position 0 and the haptic device is at an initial position $x_0 < 0$ and is pushed by a constant force towards the wall; the sample frequency is $f = 25 \text{ Hz}$. We can see from Fig.6 that the position of the haptic device increases until it meets the wall; when the haptic device gets in touch with the virtual wall it stops when the force applied by the virtual wall balances the force applied to the system by the human operator. A stable behavior is achieved even if the sample time is quite big.

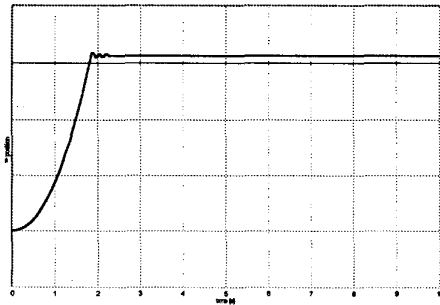


Figure 6: Virtual Wall. Position of the haptic device

7 Conclusions and Future Work

The paper has presented a novel way to approach discrete time passive control of continuous time systems like haptic devices. For haptic devices, once a physical time continuous model of a virtual environment is available, a port-Hamiltonian description can be easily calculated. Then, an energetically consistent time discrete equivalent can be realized using the techniques presented in Sec.3. Eventually, using a proper definition of the discrete time flow as introduced in Eq.(11), it is possible to obtain an EXACT balance in the flow of energy between the real physical continuous time system and the virtual environment just

described independently of the sampling time T used. Eventually, once the small discrepancy of the energy changes of the discrete time system are solved either by a clever energy book-keeping or by an electronic continuous time circuit, a passive system is obtained which can be safely used as a haptic device. The system with which the human will interact as shown in Fig.1 will be passive independently of the sampling time T used. Future work should investigate whether it is possible to geometrically define an affine connection which can be used to perform the extrapolation operations which have been done in the paper considering Euclidean coordinates. An experimental verification will be also performed in the next future.

Acknowledgments

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